

2.1 Logarithmic & Exponential Functions

Question Paper

Course	CIEA Level Maths
Section	2. Logs & Exponentials
Topic	2.1 Logarithmic & Exponential Functions
Difficulty	Very Hard

Time allowed: 60
Score: /50
Percentage: /100

Question 1a

(a) On the same axes, sketch the graphs of $y = 0.3^x$ and $y = 0.5^x$.

Label any points of intersection with the coordinate axes.

Write down the equations of any asymptotes.

[4 marks]

Question 1b

(b) Write down an equation for the graph that is a reflection of $y = 0.5^x$ in the y -axis.

[1 mark]

Question 2a

(a) Sketch the graph of $y = 0.2^{-x}$, stating whether this graph indicates exponential growth or exponential decay.

[3 marks]

Question 2b

(b) Find the value of x when $y = 625$.

[1 mark]

Question 3a

(a) Without using a calculator, evaluate $\log_4 128$.
Show each stage of your solution carefully.

[2 marks]

Question 3b

(b) Evaluate $\frac{3\log_6 216 - \ln e^5 + 4^{\log_5 625}}{\log 10000}$.

[2 marks]

Question 4

Solve $3^{2(x+1)} + 3 = 28(3^x)$.

[4 marks]

Question 5

Find two values of x for which $\log(x^2) = (\log x)^2$ is true.

[4 marks]

Question 6a

(a) Sketch the graph of $y = 4e^x$ for $x \geq 0$.

Label any points of intersection with the coordinate axes.

Write down the equations of any asymptotes.

[3 marks]

Question 6b

(b) Find the gradient of $y = 4e^x$ at the point where $x = 3$, giving your answer correct to 3 significant figures.

[1 mark]

Question 6c

(c) The population growth of population, P , at time, t years, is modelled by the equation

$$P = 4e^t.$$

Write down the initial population.

[1 mark]

Question 7a

The function $f(x)$ is defined by $f(x) = 5e^{3x}$ for $x \in \mathbb{R}$.

(a) Find $f(4x)$.

[2 marks]

Question 7b

(b) Find $f'(5x)$.

[2 marks]

Question 8a

(a) Find the gradient of the curve $y = \frac{1}{a}e^{-bx}$, where a and b are constants.

[1 mark]

Question 8b

(b) State a condition on b to ensure y represents exponential decay.

[1 mark]

Question 8c

(c) At the point $(0, a)$ the gradient is 10. Find y in terms of a (and x) only.

[2 marks]

Question 9a

A particle is travelling with velocity, $v \text{ ms}^{-1}$, at time t seconds.

The velocity of the particle is modelled $v = 0.3e^{kt}$, where k is a constant.

(a) Write down the initial velocity of the particle.

[1 mark]

Question 9b

(b) Find an expression (in terms of k and t) for the acceleration of the particle.

[2 marks]

Question 9c

(c) After 12 seconds the velocity of the particle is 0.9 ms^{-1} .

Find the value of k , giving your answer to 3 significant figures.

[3 marks]

Question 9d

(d) State a problem with the model for large values of t .

[1 mark]

Question 10

Solve $(e^x - e^{-x})^2 = 0$.

[4 marks]

Question 11

Solve the equation $2e^{3x} - 11e^{2x} + 12e^x = 0$, giving answers to 3 significant figures where appropriate.

[5 marks]